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CORRELATIONS AMONG THE PARAMETERS OF THE SPHERICAL MODEL FOR ECLIPSING BINARIES

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ABSTRACT

Correlation coefficients have been computed to investigate the parameters used to describe the spherical model of an eclipsing binary system. Regions in parameter hyperspace have been identified where strong correlations exist and, by implication, the solution determinacy is low. The results are presented in tabular form for a large number of system configurations.

INTRODUCTION

Russell and Merrill (1952) have pointed out that indeterminacy in the elements of eclipsing binaries can persist even though high quality observations are available and regardless of the formal method employed to solve for the elements. Such a situation exists for the case of shallow partial eclipses; large changes in the ratio of the radii result in only minor changes in the shapes of the eclipses. Another example is the low determinacy of the limb-darkening coefficients for partial eclipses. In Russell and Merrill's discussion, generalization of the determinacy of a solution is based on the size of the angle formed at the intersection of the shape and depth relations. The shallower the intersection the

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lower the determinacy. A number of troublesome situations are described by Russell and Merrill and these results will not be repeated here. Kopal (1959) has discussed this problem also and points out that the rate of convergence is related to the determinacy of the elements and that a solution depends upon the nature of the eclipses. Again, shallow partials produce low determinacy.

It is difficult to express the determinacy in a closed analytic form and for this reason a numerical approach is used in the present work. Rather than considering the weight or determinacy of the parameters directly, it was found more convenient to employ the correlations among the parameters as indicators of the reliability of the elements. The primary objective of this work then is to identify those regions in parameter hyperspace where the correlations are most serious and, by implication, the determinacy the poorest. In practice one is required to use data from both eclipses for partials due to the ill-conditioning of the eclipse relations. To simulate this practical aspect, the correlation coefficients have been computed with model data from both eclipses. Preliminary results indicated that the maximum geometric eclipse depth, p is a suitable choice for the independent variable in order to demonstrate the trends in the correlations in an efficient and meaningful way. The matrix of correlation coefficients is found for a range of geometric depths at grid points representing a wide range of eclipsing configurations. Only spherical stars with no radiative interactions are treated.

Method

A program for calculating the light and light loss during eclipses of spherical stars was written by the authors for the IBM 360 computer at GSFC. Although in its most general form eccentric orbits and non-linear limb-darkening can be treated, the eccentricity was set to zero and the non-linear terms for the darkening were suppressed in this work. In this initial treatment of the problem it was decided to express the model in terms of the most basic set of parameters. These are given below.

rg, rs - radii of the larger and smaller stars, respectively, in units of the semi-major axis.

i - inclination of the orbital plane relative to the plane of the sky.

ug, us - linear limb-darkening coefficients of the respective stars.

R - ratio of surface brightnesses, the cooler to the hotter star.

The light of the system is given by the following:

$$\mathcal{L} = 1 - \frac{1}{L} \iint I d \sigma$$

where the limits of the integral refer to the eclipsed portion of the eclipsed star, d^σ is the elemental surface area, L is the sum of the light of the uneclipsed components, and $I = I_0 \ (1-u+u\mu) \ \text{expresses} \ \text{the conventional linear limb-darkening relation.}$

In an orthogonal system this equation can be integrated analytically over one dummy variable. Tests are made within the program to determine the limits for the first integration as a function of the geometry of the eclipse appropriate for the given mean anomaly. Then the resulting expression is evaluated numerically by Gaussian quadratures using double precision arithmetic throughout. A ten point Gaussian form is used. The accuracy is such to reproduce the tabulated α values given by Russell and Merrill. At each grid point, and maximum geometric depth the light is calculated for 40 equidistant phase values between minimum and external tangency for the ascending branch of both eclipses. Of course $\mathcal{J} \equiv 1$ outside of eclipses.

The correlation coefficients, ρ_{ij} , of the six parameter models are given in terms of the elements, E_{ij} , of the moment matrix as $\rho_{ij} = E_{ij}/(E_{ii}E_{jj})^{1/2}$. In turn, the moment matrix is defined as twice the inverse of the matrix of the second partial derivatives R_{ij} where, in terms of the light, \mathcal{A} , $R_{ij} = \frac{\partial^2 \mathcal{A}}{\partial \theta_i} (\theta_1, \dots \theta_6)/\partial \theta_j$. The θ_i 's are the six parameters defined above. In order to compute the derivatives, a total of 73 distinct models must

be computed for each grid point at each geometric depth; one to define the run of the light curve for the parameters given at the grid location and the other 72 for the light curve variations induced by the cross term perturbation of the parameters. Subsections of the subroutine STEPIT, written by J. Chandler (1965), were used to organize these computations and perform the matrix inversion. The results appear in Tables 1-5. Unless noted the computations are for $r_g = 0.25$ and equal limb-darkenings, $u_g = u_s = 0.6$, but Tables 4 and 5 show results for significant changes in these base parameters. The explanation of the tabulated material is as follows:

- Column 1 ratio of the radii, occultations have k > 1.0.
 - 2 geometric depth, p
 - 3-7 correlation of r_g with r_s , i, u_g , u_s , and R.
 - 8-11 correlation of r_s with i, u_g , u_s and R.
 - 12-14 correlation of i with u_g , u_s and R.
 - 15-16 correlation of u_g with u_s , and R.
 - 17 correlation of us with R.

A row of zero values means that no computations were made for that particular geometric depth. A negative sign means anticorrelation.

Discussion

No attempt will be made to discuss the form or significance of all the correlation trends apparent in this material or to generalize them. The intention of this work is to provide a framework within which the analysis of individual solutions can be assessed. High correlations imply poor convergence and certainly indicate anomalous error assignments for the solution parameters. For perspective, the commonly accepted rule is that $\rho < 1.5$ implies no correlation while $\rho > 0.8$ implies high correlation. The generally conceded increase in the indeterminacy for shallow partial eclipses is evident and is quantified in this work. The high determinacy for total eclipses is verified except that a moderately strong correlation between $u_{\rm S}$ and R

appears to exist. As R approaches zero, the main parameters r_g , r_{sj} and i become strongly correlated. Troublesome correlations also appear in partial occultations for internally tangent configurations.

From a different point of view these results tend to reaffirm the assertion made by Kopal that a solution is not complete unless the errors of the elements are evaluated. As a future investigation, it would appear desirable to study selected combinations of parameters which would minimize the correlations or remove the skewness of the probability error ellipse and thereby improve the determinacy of the solution.

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- Chandler, J. P. 1965, Quantum Chemistry Program Exchange, Indiana University, Bloomington, Indiana.
- Kopal, Z. 1959, Close Binary Systems, The International Astrophysics Series, Vol. 5, J. Wiley and Sons, Inc., New York.
- Russell, H. N. and Merrill, J. E. 1952, Contr. Princeton Univ. 26.

CAPTIONS

- Table 1 Correlation coefficients for a light ratio of 0.9.
- Table 2 Correlation coefficients for a light ratio of 0.6.
- Table 3 Correlation coefficients for a light ratio of 0.2.
- Table 4 Correlation coefficients for a range in the size of the primary star but with k = 1.0.
- Table 5 Correlation coefficients when the limb-darkenings differ from 0.6.

Table 1

CORRELATIONS FOR A LIGHT RATIO = 0.9

	1,56		-C+3			-0.01		_	0.25	0.25	0.03	-0.31	-0,28	-0.09	-0.21	0,99
	1 • 52		- .0 _● 55		0.20	0 • 10	-0.87	-0 • 29	9.37	0.36	0.05	-0.42	-0.39	- 0 . C8	- 0 ₀ 22	0,98
	1 • 48		- Co 44		0.19	0 • 05	-0.89	-0 • 35	0.36	0.37	0.14	-0.42	-0.40	-0.07	- 0e 27	0, 97
	1 • 44	0.47	-0.61	0, 78	0.45	0.11	-0.90	0.21	0.25	0.12	-0.40	-0.37	-0.18	0.36	- C. 10	0,89
	1.40	0 - 14	-0 ₀ 35	0.83	0.82		-0.94		0.14	0.02	-0.29	-0.32	-0.21	0.72	0.20	0.82
0.50	1 • 36	-0.68	0,64	0, 73	0.87	0 • 69	-1.00	-0.06	-0.28	-0.48	0.04	0.24	0.42	0.90	0.33	0.71
0.50	1 • 32	-0,43	0.93	0.50	0.74	0.03	-0.99	-0.33	-0.47	-0.02	0.39	0.50	-0.05		- 0 ₀ 34	0.09
0.50	1 • 28	0.0	0.0	0,0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
											- • •		•••		•••	4.0
0.00	1.55	-0.07	-0.21	0.70	-0.01	-0 a 25	-0.60	-0 a 37	0.30	0.36	-0-23	-0-43	-0.26	-0.11	-0. AE	0, 92
090	1.52	0.12	-0.61	0, 83		-0.21			0.31		-C. 76				- 0 ₀ 35	0,73
	1 • 48		-0.54	-	0.62		-0.86	-	0.50		-0.86				~0.09	0.69
	1 . 44		-0.68		0.84		-0.86	0 • 66	0.58		-0.92					
	1.40		-0.79		0.92		-0.73		0.44							0,67
	1.36		-C. 68		0.92					0.16	-0.94 -0.86	-0.92	-0.44			0.66
0.80			-0.26		0.83		-0.50							0 • 93		0,54
2.50			C. 27	-							-0.50				-0.20	0, 32
) 5 C C	1920	- O & 7 c.	U	Us 73	0.50	-0.37	-0.75	-0 - 34	-0.06	G • 4A	C•22	-0.11	-0.46	0.67	-0.54	0, 24
1.00	1 54	0 37		0 70	0 05	• ••	•									
			-C.11		-0.25				0.19					-0.37	-0.80	0.84
1.00			-0.30	0.34		-0.07		0.03	C.18		-0.68		-0.01	0.15	-0.61	0.67
1.00		-0.42	-	0.48	0.50		-0.49	0 • 30	0.33	-0.10	-0.89	-0.86	0.0	0.65	- 0e 38	0.45
1.00			− 0• 75	0, 74	0.77		-0.74	0.72			-0.97		0.0	0.51	-C-18	0,23
1.00			-0.91	Co 91	C• 92		-0.91	0 • 90	0.88	-0.08	-0.99	-0.99	0.0	0.98	- 0e 08	0,11
1.00		0.13	-0 ₀ 77	0. 76	0.80	0.10	-0.76	0.74	0.69	-0.10	-0.98	-0.97	0.0	0.95	-0-13	0.18
1.00	1 • 32	- C • 40	_	0.55	0.61	0 • 05	-0.55	0.47	0.39	-0.04	-0.92	-0.90	0.0	0.87	-0.22	0.29
1.00	1.28	-0.62	-C.38	0.47	0.44	-0 · 14	-C.45	0.27	0.29	0.15	-0.74	-0.72	0.0	0.76	-0.32	0.36
1.25	1.56	-0.04	-0.57	0, 38	-0.39	-0·43	-0.29	-0.01	0.64	0.25	-0.48	-0.26	0.28	-0.16	-0-91	0,52
1.25	1.52	0.13	-0.62	0,38	0.11	-0.23	-0.61	0.39	0.78	0.22	-0.59	-0.76	0.0		- C. 72	0.41
1.25	1 • 48	0.16	-0.85	0.58	0.58	-0.15	-0.54	0.58	0.68		-0.79		0.17	-	-0.67	0.14
1.25	1 . 44	0.34	-0.83	0, 70	0.73	-0.25	-0.73	0.83			-0.92		0.41		-0.66	
1.25	1.40	C • 39	-0.80	Co 65	0.69	-0.26	-0.86	0.93			-0.96		0.50		-0.64	
1.25	1 • 36	-C.26	-0.44	C. 13	0.19	0 . 04	-0.75	0.89			-0.91		0.29		-0.46	
1.25	1.32	-0.62	-0.45	-0.05	-0.07			0.75			-0.72		0.19		- Co 27	
25	1 . 28	- C. 72	-Co 67		-0.23			0.42			-0.29		0.47		-0.25	
				-					•••	3 . . L	0.27	0413	0.41	V & U 9	-0.25	0.02
2.00	1 • 56	0 a 3.8	-0 ₆ 75	0.36	-0.38	-0.38	-0.50	0.10	0.56	-0.03	-0.43	0 00				
2.00			-0.85		-0.30			0.29			-0.58			-0.20		0.30
2.00			-0.37		-0.34			3 • 25			-	. •		-0.19		0,30
2.00			-0.88	0,32		-0 • 21					-0.57			-C.19		0, 35
2.00			- Ca 91	0.27				0.45			-0.45		0.28	-	- C. 91	0,19
2.00				- 0° 21	-	-0.13		0.84			-C.51	-	0.39		- Co 94	
						0.41	0.55	0.67		-0.67		-0.04			- Ce 57	
2.00				- C. 47			0.92	75 و 0	0.61	0.01	0.49	0.41	0.13	0.91	-C. 02	0,39
2000	1020	O ₃ C	CaG	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)

See text for explanation of tabular material.

Table 2

CORRELATIONS FOR A LIGHT RATIO = 0.6

0.50 1.56 C.25 -C.24 0.73 0.03 -0.06 -0.82 -0.36 0.17 0.18 0.20 -0.22 -0.22 -0.07 -0.19 0.99 0.50 1.52 0.33 - 0.33 0.62 0.09 0.0 -0.91 -0.42 0.29 0.30 0.31 +0.33 +0.33 +0.33 +0.12 +0.240.50 1.48 C.10 -0.10 0.65 0.07 -0.07 -0.94 -0.55 0.29 0.34 0.46 -0.33 -0.37 -0.12 -0.30 0.50 1.44 0.40 -0.48 C. 88 0.44 0.07 -0.93 0.19 0.17 0.06 -0.32 -0.27 -0.11 0.41 -0.03 0.50 1.40 -0.02 -0.08 0.89 0.79 0.45 -0.97 0.02 -0.04 -0.09 -0.06 -0.05 -0.02 0.74 0.28 0.50 1.36 -0.77 0.77 0.71 0.86 0.70 +1.00 -0.14 -0.41 -0.58 0.16 0.41 0.56 0.87 0.34 0.77 0.50 1.32 -0.95 0.95 0.57 0.72 0.09 -1.00 -0.34 -0.51 -0.09 0.39 0.55 0.05 0.87 -0.23 0.25 0.5C 1.28 0 • C 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.80 1.56 -0.17 -0.01 0.79 0.0 -0.25 -0.65 -0.39 0.15 0.24 -0.08 -0.33 -0.23 -0.03 -0.36 0.93 0.80 1.52 0.09 -0.62 0.91 0.46 -0.20 -0.62 0.11 0.19 0.07 -0.76 -0.54 0.04 0.48 -0.26 0.80 1.48 C.19 -0.55 C.85 0.67 0.13 -0.87 C.52 0.40 0.02 -C.83 -0.68 -0.11 0.70 0.0 0.71 0.80 1.44 2.68 -0.56 0.87 0.83 0.38 -0.85 0.47 0.37 0.03 -0.84 -0.75 -0.24 0.86 0.24 0.70 0.80 1.40 -0.19 -0.54 0.91 0.90 0.41 -0.71 0.16 0.08 -0.09 -0.79 -0.73 -0.25 0.92 0.29 0.89 1.36 -0.66 -0.30 0.94 0.93 0.30 -0.51 -0.41 -0.44 -0.18 -0.53 -0.50 -0.15 0.94 0.20 0.52 0.20 1.32 -0.64 0.34 0.94 0.91 -0.08 -0.78 -0.67 -0.63 0.14 0.16 0.11 -0.19 0.92 -0.17 0.22 0.80 1.28 -0.80 0.69 0.90 0.70 -0.53 -0.92 -0.68 -0.41 0.59 0.63 0.35 -0.58 0.77 -0.58 0.07 1.00 1.056 -0.29 -0.11 0.61 -0.08 -0.34 -0.09 -0.45 -0.05 0.19 -0.26 -0.30 -0.05 -0.15 -0.67 1.00 1.52 -0.64 -0.30 0.66 0.33 -0.13 -0.26 -0.25 -0.08 0.06 -0.67 -0.61 -0.03 0.32 -0.44 1.00 1.48 -0.46 -0.52 0.73 0.63 0.01 -0.44 0.11 0.09 -0.06 -0.88 -0.80 -0.01 0.72 -0.24 1.00 1.00 0.00 -C.74 0.84 0.82 0.03 -C.70 0.53 0.48 -0.09 -C.96 -0.93 0.02 0.91 -0.13 0,28 C.39 -0.84 0.90 0.90 0.01 -0.83 0.73 0.70 -0.08 -0.98 -0.96 0.03 0.96 -0.10 1.00 1.40 0.16 1.00 1.36 -0.04 -0.68 0.83 0.83 0.01 -0.70 0.48 0.43 -0.03 -0.95 -0.91 0.01 0.94 -0.11 1.00 1.032 -0.50 -0.43 0.77 0.74 -0.07 -0.56 0.10 0.08 0.07 -0.83 -0.77 0.0 0.88 -0.16 0.32 1.00 1.28 -0.69 -0.14 0.76 0.62 -0.24 -0.59 -0.15 -0.05 0.26 -0.53 -0.51 -0.05 0.79 -0.27 0.36 0.01 -0.49 0.48 -0.40 -0.52 -0.43 0.07 0.52 0.18 -0.55 -0.34 0.26 -0.12 -0.86 0.58 1.25 1.56 1.25 1.52 0.13 -0.56 0.50 0.07 -0.34 -0.64 0.37 0.68 0.20 -0.68 -0.76 0.0 0.36 -0.62 0.49 0.10 -0.65 0.75 0.58 -0.32 -0.51 0.48 0.57 -0.06 -0.88 -0.85 0.26 0.67 -0.60 0.18 1.25 1.48 1.25 1.44 0.44 -0.90 0.85 0.82 -0.45 -0.76 0.78 0.77 -0.39 -0.97 -0.95 0.49 0.91 -0.62 -0.24 1.25 1.40 0.61 -0.85 0.85 0.85 -0.44 -0.90 0.91 0.90 -0.46 -0.99 -0.98 0.51 0.96 -0.57 -0.34 1.25 1.36 -0.69 -0.57 0.52 0.49 -0.13 -0.76 0.77 0.73 -0.07 -0.96 -0.90 0.17 0.90 -0.25 0.19 1.25 1.32 -C.56 -0.42 C.37 C.23 -0.24 -0.49 O.48 O.50 O.22 -0.84 -0.65 O.14 O.76 -0.16 O.51 1.25 1.28 -0.74 -0.48 0.46 -0.08 -0.54 -0.08 0.10 0.49 0.53 -0.42 0.03 0.41 0.53 -0.24 0.69 2.00 1.56 0.39 -0.66 0.42 -0.41 -0.42 -0.63 0.19 0.48 -0.10 -0.47 -0.11 0.42 -0.17 -0.98 0.32 0.59 -0.30 0.57 -0.25 -0.55 -0.77 0.40 0.45 -0.30 -0.62 -0.16 0.56 -0.09 -0.98 0.26 2.00 1.52 2.00 1.48 0.52 -0.81 0.55 -0.26 -0.54 -0.71 0.37 0.44 -0.25 -0.61 -0.10 0.54 -0.10 -0.96 0.34 2.00 1.44 0.57 -0.84 0.46 0.19 -0.30 -0.78 0.53 0.62 -0.19 -0.59 -0.44 0.34 0.24 -0.84 0.31 2.00 1.40 0.43 -0.83 0,50 0.37 -0.29 -0.70 0.85 0.67 -0.48 -0.71 -0.42 0.53 0.60 -0.71 0.13 2.00 1.36 -0.54 -0.95 -0.08 0.06 0.26 0.28 0.86 0.72 -0.60 -0.18 -0.25 0.0 0.91 -0.58 -0.19 2.00 1.32 -0.92 -0.97 -0.49 -0.39 -0.13 0063 0009 0045 0043 0031 0090 0009 0051 0.86 0.77 2,00 1,28 0.0 0 0 0 0 0 0 0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 (1) (2) (3) (4)(5) (6) (7) (8) (9) (10) (11) (12) (13) (14) (15) (16) (17)

See text for explanation of tabular material.

Table 3

CORRELATIONS FOR A LIGHT RATIC = 0.2

0.50	1 • 56	-0.17	C 3 3 6	0- 00	0.07	-0 - 00	-0.00	-0 =4	2 21	0 07	0 67	0 07		2 22		• •
0.50		0.04	0.03		-0.C7								-0.10			0.98
	1.48	0 . C 6	C. 04		-0.06					0.31			-0.33			0,99
0.50			-C.25	0.95	0.25		-0.99		0.25	0 • 27	_	-	-0.31			0.99
	1.40	-0.31	Co 37	0, 95	0.67		-		0.25		-0.08			0.20	0.0	0.97
0.50		-0.96	0.96				-0.99				0.39	0.36	•	0.66	0 22	0 86
	1.32	-	-	0.66	0.84		-1.00				0.46	0.76	-	0.71	0.42	0.93
-	_	0 · C	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0 • 0	0.0	0.0	0.0	0 • C	0.0	0.0
0.50	1.28	0 • C	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0• 0	0.0
0.80	1 • 56	-0.48	0.57	Co 92	0.14	-0-13	-C•79	-0 - 50	-0-10	0.00	0.40	-0-01	-0.13	0.15	- 0. 15	0.93
	1 . 52		-C.69	0, 96			-0.96						0.06	0	-0.21	0.87
	1.48		-0.64	C, 97			-0.91				-0.77	-			- Co 01	0.58
0.80			-0o11	0, 95		-	-0.93	-			-0.38				0.02	0, 55
	1.40	-0,50	C. 25	0. 94			-0.95	-							- 0 ₀ 03	0, 57
	1.36	-0.75	0.51	0. 95			-0.98				0.44		-		-0.16	0, 41
	1.32	-0.95	0.94	0,99			-1.00								-0.61	
-	1.28	-0,93		- Ca 03			-1.00				-0.31					
000		0,,,5	00 70	0000	0002	0014	-1000	0.00	0015	0 9 1 3	-0.51	-0013	- 70 1 3	0.00	-0e 04	0.75
1.00	1,56	-0.82	0.82	0.96	0.48	0.03	-0.73	-0.81	-C.57	-0.19	0.76	0.25	-0-10	0 - 38	-0.09	0.88
1.00	1.52	-0.74	. 0.05	0, 95			-0.43				-0 · 15				- 0a 32	0.38
1.00	1.48	-0.61	-0.15	0. 94			-0.60				-0.42			_	- 0 · 31	0,28
1.00	1 . 44	-0.40	-0.25	0, 94			-0.75				-0.55		-		- 0e 34	0.16
1.00	1-40		-0.12	0. 92			-0.84		0.02		-0.47				- 0 ₀ 34	0.16
1.00		-0.53	0.15	0.92			-0.88				-0.20				- 0e 37	0.19
1.00		-0.75		0, 94		_	-0.93				0.21				- 0e 45	0.12
1.00	1.28	-0.89	0.80	O _a 96			-0.97				0.68				- 0. 64	
												0.44	0.00	0401	00 04	-000
1.25	1.56	-0.06	-0.19	0.71	-0,18	-0.55	-0.58	0.07	0.27	0.13	-0.49	-0.51	0.07	0.14	-0.51	0.63
1.25	1 • 52	-0.14	-0.20	0, 73	0.07	-0 • 52	-0e70	0.13	0.49	0.23	-0.61	-0.76	-0.0€		-0.45	0.53
1.25	1.48	-0.88	-0.68	0.95	0.77	-0 • 37	0.32	-C • 77	-0.61	0.29	-0.79	-0.79	0.12	0.75	- C. 44	0.25
1.25	1.44	-0.88	-0.97	1.00	0.98	-0.98	0.74	-0. 85	-0.81	0.88	-0.98	-0.99	0.94	0.59	-0.98	-0.94
1.25	1 = 40	-0.57	- 0a 52	0, 95	0.80	-0 a 84	-0.38	-C • 30	-0.08	0.50	-0.75	-0.83	0.33	0.51	-0.76	-0.43
1.25	1.36	-0 .7 9	0.01	0.93	0.65	-0 e 83	-0.61	-0 •54	-0.15	0.77	-0.31	-0.53	-0.16	0.82	- 0. 72	-0.21
1.25	1.32	-0.96	0.66	0.98	0.68	-0 • 9a	-0.85	-0 -88	-0.52	0.93	0.54	0.17	-0.68	0.78	-0.89	-0.42
1.25	1 . 28	-0.93	0.90	- 0e 11	-0a17	-0.29	-0.99	0.46	0 • 37	0.27	-0.49	-0.34	-0,20		0.07	
																-
2.00			-0.50	00 34	-0.40	-0.31	-0.80	0.28	0.43	-0.04	-C • 41	+0.45	0.18	0.34	- 0e 57	0.32
2.00			- Ce 35	Oa 43	-0.13			0.55	0.55	-0.20	-0.63	-0.65	0.29	0,55	-0.64	0.15
2.00		0.37	-0.43	0,49	0.01	-0 o 35	-C.91	0 • 59	0.65	-0 0 1 7	-0.68	-0.66	0.27	0.65	- 0. 52	0,21
2.00	1 • 44	0.67	-0.71	0.75	0 ₀ 59	-0 • 33	-C.97	0 .85	0.83	-0.22	-0,58	-0.68	0.29	0.90	-0.31	0.05
2.00	1 • 4 C	0.50	-0.93	0, 39	0.67	-0.53	-0.76	0.81	C.57	-0.54	-0.97	-0.68	0.65	0.73	-0.62	0,07
2.00	1,36	-0.44	-0.23	0.21	0 0 1 1	-0,18	-0 ₀ 75	0 • 77	0.50	-0.10	-0.92	-0.56	0.43	0.79	- Co 15	0,49
2.00	1 • 32	0 • C	0.0	0,0	0.0	0.0	C • O	0.0	0.0	0.0	0.0	0.0	0.0	0 • C	0.0	0.0
2.00	1 • 28	0.0	C e O	0.0	0.0	0 . 0	0.0	0.0	0.0	0.0	0 • 0	0.0	0.0	0.0	C. 0	2.0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(3)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
See 1	text f	or exp	lanati	on of	tabula	r mate:	rial.									

Table 4

CORRELATIONS AS A FCN OF RAD FRIM

RAD PRIM	= C • 15													
1.00 1.56	-0.20 -0.2	1 0,48 -	C.40 -	-0 - 4.9	-0 -1 7	-0 • 42	C • 3 =	0.43	-0.34	-0 • 29	0.02	-0.46	-0.84	0.8€
1.00 1.52	0.10 -0.7	o c. 68	0.55 -	0 - 11	-0.67	0.50	0.57	0.05	-0.87	-0.85	0.02	0.60	-0.43	0.45
1.00 1.48	-0.78 -C.2	a 0,25	0.58	0 • 67	-0.37	0.38	0.02	-0.67	-0.98	-0.92	0.02	0.87	- C. 16	0.35
1000 1044	0.27 -0.8	1 0077	0.82	0 - 14	-0.78	0.75	0.67	-0.10	-0.95	-0-54	-0.03	0.90	-0.17	0,28
1.00 1.40	-0.56 -0.4	3 0,46	0.40 -	-0.12	-0.45	0 • 23	0.27	0.14	-0.56	-0.54	-0.01	0.59	- Ce 41	0.48
1.00 1.36	-0.92 0.2	7 C. 21	0.17 -	0 • 32	-C.54	0.14	0.16	0.32	-0.31	-0.30	-0.10	0 • 86	- 0∙23	0,28
1.00 1.32	-0.89 -C.0	8 0, 06 -	0.17 -	C • 42	-0.2€	-0.20	0.03	0.40	0.84	0.82	-0.01	0.79	- Co 31	0.34
1.00 1.28	-0.30 -0.5	1 -0,33 -	0.56 -	0 • 24	-0.67	-0.64	-0.35	0.25	0•€4	0.79	- 0.05	0.40	- Co 56	0.54
RAD PRIM	= C•25													
1.00 1.56	-0:37 -C:1	1 0,32 -	0.25 -	-0.29	-0.11	-0 • 32	0.19	0.26	-0.27	-0.28	-0-01	-0-37	- 0- 80	0.84
1.00 1.52	-0.61 -0.3		0.15 -			-		-	-C.68				-0.61	0.67
1,00 1,48	-0.42 -0.5	- · ·			-Ce49	0 • 39			-0.89			0.65	-0.39	0.45
1.00 1.44	0.15 -0.7				-0.74	0.72	0.66	-0.13	-C 97	-0.96	0.0	0.91	-0.18	0,23
1.00 1.40	0.65 -0.9				-0.91	0.90	C.88	-0.08	-0.99	-0.99	0.0	0.98	- 0. C8	0.11
1.00 1.36	0.18 -0.7		0.80	0.10	-0.76	0.74	0.69	-0.10	-0.98	-0.97	0.0	0.95	-C.13	0.18
1.00 1.32	-0.40 -0.5	55 و 4	C. 61	0.05	-0.55	0.47	0.39	-0.04	-0.92	-0.50	0.0	C • 87	-0.22	0.29
1.00 1.28	-0.62 -0.3		0.44 -	-0.14	-0045	0.27	0 • 29	0 • 15	-0 • 74	-0.72	0.0	0.76	- 0 ₀ 32	0.36
RAD PPIM	= 0.35													
1.00 1.56	-0.50 -0.0	6 0,27 -	·0•13 -	-0 • 21	-0.08	-0 - 28	C.12	0.18	-0 a 22	-0.24	-0.02	-0.35	- Ce 79	0, 83
1.00 1.52	-C . 74 -0.1	6 0 , 24	C. 08 -	-0 • 02	-0.19	-0 • 07	0.00	0.01	-0 • 57	-0.55	-0.01	-0.C1	- 0•67	0.74
1.00 1.48	-0.76 -0.2	6 0,25	0.31	0.15	-0.27	0.15	0 • 0 5	-0.15	-0.77	-C.74	- 0 ₀ 0 1	0 • 36	- C. 51	0, 60
1.00 1.44	-0,66 -0,3	8 0,35	0,47	0 • 23	-0.38	0.32	0.17	-0.23	-0 • 8 ¢	-0.86	- 0.01	0.67	- O. 35	0.45
1.00 1.40	-0.43 -0.5	2 0,50	0.60	0.24	-C.52	0.49	0.37	-0.24	-0.96	-C.94	-0.01	0.85	- 0 ₀ 22	0,31
1.000 1.35	-0.09 -0.6	7 0,66	0.72	0.20	-0.57	0.65	0.58	-0.20	-0.98	-0•¢7	0.0		- Co 14	0,20
1.00 1.32	0.07 -0.7	2 C, 7 2	0.76	0 • 15	0.74	0.72	0.67	-0.15	-0.99	-0.98	0.0	0.96	- Co 11	0, 15
1.00 1.28	-C.15 -C.6	3 0, 64	0.69	0.12	-0.67	0.64	0.57	-0.12	-0.98	-0.97	0.01	0 • 95	- Co 13	0,17

(3) (4) (5) (6) (7) (8) (9) (10) (11) (12) (13) (14) (15) (16) (17)

See text for explanation of tabular material.

(1) (2)

CORRELATIONS WITH LIME DARK VAR

LIMB DARK = $\mu_{\text{eclipsed}} = 0.1$, $\mu_{\text{eclipsing}} = 0.6$ 1.000 1.56 -0.37 -0.47 0.95 0.71 -0.31 0.23 -0.51 -0.48 0.11 -0.52 -0.49 0.10 0.74 -0.34 0.35 1.00 1.052 -0.66 0.35 0.91 0.62 -0.27 -0.63 -0.59 -0.44 0.17 0.07 -0.08 -0.15 0.67 -0.29 0.48 1000 1048 -0064 0014 0091 0068 -0033 -0075 -0037 -0025 0022 -0022 -0030 -0012 0077 -0032 0033 1000 1044 -0040 -0050 0088 0069 -0034 -0086 0002 0011 0020 -0049 -0052 -0008 0083 -0029 0027 1a00 1a40 -0.35 Ca02 0.83 0.62 -0.32 -0.93 0.18 Ca27 0.23 -0.50 -0.55 -0.14 0.82 -0.23 0.35 1,00 1,35 -0,49 0,23 0.51 0.54 -0.33 -0.95 0.05 0.20 0.29 -0.32 -0.41 -0.21 0.77 -0.22 0.43 1.00 1.32 -C.63 0.44 0.84 0.52 -0.35 -0.96 -0.18 0.05 0.36 -C.02 -0.19 -0.28 0.72 -0.25 0.46 1000 1028 -0075 Co55 9067 0050 -0044 -0097 -0043 -0010 0046 0038 0009 -0038 0069 -0033 0043 LIMB CARK = $\mu_{\text{eclipsed}} = \mu_{\text{eclipsing}} = 0.1$ 1.000 1.000 -0.20 -0.10 0.30 -0.29 -0.37 -0.00 -0.36 0.20 0.32 -0.23 -0.23 0.0 -0.39 -0.82 0.84 1060 1052 -0072 -0021 0039 0001 -0027 -0024 -0013 0022 0025 -0063 -0062 -0001 0009 -0065 0069 1.000 1.045 +0.059 +0.41 0.45 0.35 +0.14 +0.41 0.22 0.33 0.12 +0.67 +0.65 0.0 0 • 62 - 0 • 42 0 • 45 1.00 1.00 1.00 -0.07 -0.67 0.69 0.67 -0.05 -0.63 0.60 0.61 0.03 -0.96 -0.96 0.01 0.90 -0.22 0.23 1.00 1.40 0.30 -0.79 0.80 0.80 -0.01 -0.01 0.77 0.77 -0.01 -0.98 -0.98 0.01 0.96 -0.15 0.15 1.00 1.36 -0.06 -0.65 0.67 0.68 0.02 -0.70 0.64 0.62 -0.03 -0.96 -0.95 0.0 0.52 -0.18 0.22 1.00 1.32 -0.43 -0.43 0.53 0.53 0.01 -0.57 0.45 0.43 -0.01 -0.89 -0.87 0.0 0.84 -0.25 0.31 1.00 1.28 -0.61 -C.35 0.46 C.39 -0.09 -0.49 0.28 0.33 0.10 -0.68 -0.66 0.0 0.74 -0.33 0.39 (1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12) (13) (14) (15) (16) (17)

Table 5

See text for explanation of tabular material.